Hamiltonian operator

If we consider potential term $V(\vec{r},t)$ in study, then total energy, $E = \frac{p^2}{2m} + V(\vec{r},t)$

Then

$$\hat{E}\psi = \left(\frac{\hat{p}^2}{2m} + V\right)\psi$$

$$i\hbar\frac{\partial}{\partial t}\psi(\vec{r},t) = \left(-\frac{\hbar^2}{2m}\nabla^2 + V(\vec{r},t)\right)\psi(\vec{r},t)$$

Then

$$H = -\frac{\hbar^2\nabla^2}{2m} + V(\vec{r},t)$$ (Hamiltonian operator)

Position Probability

$$P(\vec{r},t) = \psi^*(\vec{r},t)\,\psi(\vec{r},t)$$

Position Probability density and its normalization condition; $\int|\psi(\vec{r},t)|\,dr = 1$

Expectation value of Dynamical Variables

The average value of the measurements of the quantity performed on a very large number of independent identical systems represented by the wave function, ψ.

Thus expectation value of position is given by

$$\langle r\rangle = \int r\,P(\vec{r},t)\,dr$$

$$= \int \psi^*(\vec{r},t)\,r\,\psi(\vec{r},t)\,dr$$

similarly, for 1-D

$$\langle x\rangle = \int \psi^* x \psi \, dx$$

$$\langle y\rangle = \int \psi^* y \psi \, dy$$

$$\langle z\rangle = \int \psi^* z \psi \, dz$$

The expedation value of any quanity is given by

$$\langle g(r,t)\rangle = \int \psi^*(\vec{r},t)\, g(\vec{r},t)\, \psi(\vec{r},t)\, d\vec{r}$$

similarly we can write

$$\langle E\rangle = \int \psi^* \left(-i\hbar \frac{\partial}{\partial t}\right) \psi \, dr$$

$$\langle \vec{P}\rangle = \int \psi^* \left(-i\hbar \vec{\nabla}\right) \psi \, dr$$

and its component

$$\langle P_x\rangle = -i\hbar \int \psi^* \frac{\partial}{\partial x} \psi \, dx$$

$$\langle P_y\rangle = -i\hbar \int \psi^* \frac{\partial}{\partial y} \psi \, dr$$

and

$$\langle P_z\rangle = -i\hbar \int \psi^* \frac{\partial}{\partial z} \psi \, dr$$

Scanned with CamScanner